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The analysis of buckling and post buckling in the compressed composite columns

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ABSTRACT

Purpose: The aim of the study was to analyse the work of a thin-walled C-shaped profile, made of a carbon-epoxy composite, which was subjected to unified axial compression.

Design/methodology/approach: The scope of the study included the analysis of the critical and low post-critical state by the use of numerical and experimental methods. As a result of the experimental test, performed on the physical specimen, post-critical equilibrium path had been determined, on the basis of which, with use of the adequate approximation method critical load value was defined. The next stage of the research was devoted to numerical analysis based on the finite element method. The studies were carried out on a scope of the linear analysis of the eigenvalue problem, on the basis of witch the critical value of load for mathematical model was found. The next step of the numerical tests was covering the nonlinear analysis of the low post-critical state for the model with geometrical imperfection, corresponding to the lowest form of buckling.

Findings: The result of the study was to determine the value of the critical load, on the basis of the experimentally obtained post-critical equilibrium paths of the structure, with use of two independent methods of Approximation: Koiter's method and the method of the vertical tangent. The results of the analysis were compared with the value of the critical load determined by using finite element method.

Research limitations/implications: The obtained results of study provide the important information concerning the modelling techniques of the thin-walled structures made of composite materials, while confirming the adequacy of the numerical models developed both in the calculation of eigenvalue problem, as well as non-linear static analysis in the post-critical range.

Originality/value: The research provided the necessary knowledge of the behaviour of the critical and low post-critical of the thin-walled structure made of modern orthotropic material (CFRP).

Keywords: Thin-walled structures; Stability of construction; Critical state; Composites; Finite Element Method

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

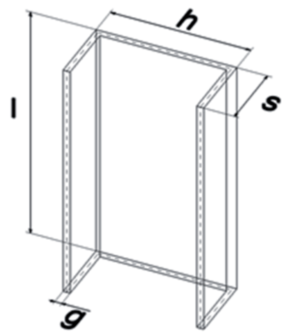
High stiffness and strength characteristics while maintaining low own weight made the thin-walled components widely used in load-bearing structures. These features make extensive use of this type of structures in multiple industries, such as aerospace and automotive. This applies in particular to thin columns with complex cross-sections, which are used as stiffening elements for hauls sheathing or bodywork. Undesirable feature associated with the exploitation of this type of construction is the possibility of buckling, even under operational loads [1-6]. When buckling of the thin-walled structure is local and elastic, it does not lead to structural damage and the construction may be able to operate safely in post-critical state [3,4,7]. Therefore, knowledge of the critical force, at which buckling of thin-walled structure occurs is a question of primary importance in the exploitation conditions. Unfortunately, the method of determining the value of the critical load for the real construction are not clear, which further complicates the process of optimal design of this type of structures. In such cases an alternative tool that allows to determine the values of the critical load are the numerical analyses using the finite element method [3,4,8-10]. The value of the critical load is determined using the linear analysis of eigenvalue problem, based on the criterion of minimum potential energy of the considered structure. Numerically determined value of the critical force may in such cases be a reliable prediction of the value because of the fact that the perfect designed structure is assumed in the calculation, disregarding the geometrical inaccuracies, occurring in the actual structures. This means that the numerical models developed for thin-walled structures, often characterized by complex shapes of cross sections should be based on the experimental results. Such procedure allows the development of adequate computer models for analysing complex issues of buckling and work of thin-walled structures in post-critical state [4,11-13].

Requirements to be met in present-day industry, caused that the thin-walled structures used for advanced aircraft structures or traditional automotive engineering materials are being replaced with modern composite materials. This is particularly true for composite laminates referred as multilayer composites [14,15], due to the construction of composite materials. These materials are primarily characterized by high strength in relation to its own weight, which determines their use in the load-bearing structures. An additional advantage of the composite material is possibility of shaping their mechanical properties by the selection of required material properties and the use of a suitable arrangement of layers of the laminate configuration. The literature on the thin-walled composite bearing structures is limited, dominated mainly by the theoretical studies, with a few publications referring the results of experimental tests.

The subject of the research carried was the analysis of the structure's critical state and the behaviour in low post-critical state of the axially compressed thin-walled column with a C-shaped cross-section, made of carbon-epoxy composite. The research involved determination of the critical load of the physical structure, and analysis of critical and post-critical state based on FEM. The studies show the methodology leading to the solution of the problem of buckling and nonlinear stability of the thin-walled components made of composite laminates.

2. The subject of research

The subject of research was a short, thin-walled column with a channel section, subjected to an axial compression. The examined structure was a typical thin wall structure, consisted of a perpendicular wall, forming a flat plate elements connected at the long edges [7,11]. The structure was made using an autoclaved technique from carbon-epoxy composite with the reference number M12/35%/UD134/AS7. The structure of the composite was consisted of 8 layers in a symmetrical arrangement relative to the median plane configuration [0/90/0/90]s. Overall dimensions of the C-shaped profile were shown in Fig. 1.



height	***h***	*80*	*mm*
width	***s***	*20*	*mm*
length	***l***	*147*	*mm*
thickness	***g***	*1.148*	*mm*

Fig. 1. Geometrical model of the C-shaped profile with dimensions

For the manufactured composite material, basic mechanical properties accordance with ISO were experimentally determined. Experimentally defined boundary and mechanical properties of the carbon-epoxy composite material enabled the development of the material model with the orthotropic properties. The mechanical properties of the test material are shown in Table 1.

Table 1.
Mechanical properties of CFRP

Tensile strength F_R, MPa	0°	1867.2
	90°	25.97
Compressive strength F_C, MPa	0°	1531
	90°	214
Shear strength F_R, MPa	± 45°	100.15
Young's modulus E, MPa	0°	130710
	90°	6360
Kirchhoff's modulus G, MPa	± 45°	4180
Poisson's ratio, -	0°	0.32

3. Methodology of the research

The researches carried out included the scope of the analysis of the critical and low post-critical compression of the thin-walled composite structure by the means of experimental and numerical methods. Experimental studies on the manufactured composite thin-walled columns enabled observation he actual behaviour of the structure in a critical state, and its operation after buckling. Conducted in parallel numerical simulations, sought to develop adequate, experimentally verified FEM models, to allow modelling of stability issues of post-critical state of the composite thin-walled structures, imitating in a faithful manner the behaviour of the actual structure.

3.1. Experimental studies

The test of the axial compression of thin-walled composite short column of C-shaped cross-section at the load range of up to 150% of the critical load (set in the numerical analyses) was performed. Experimental studies were carried out on Zwick Z100 universal testing machine, at the range of the load of 100 kN at 23°C, at a constant speed of movement of the upper cross-piece of 2 mm/min. During the tests the end of channel cross sections were freely supported, thereby ensuring the realization of the free articulated support for all the walls of the column. In order to avoid the impact of inaccuracy of the tips of the cross-sectional profile on the work of the structure, at both ends soft plastic pads were used, enabling the superficial support of end sections of the column. The column was aligned with the use of dedicated dices, enabling the correct setup of the C-shaped cross-sections in relation to the pivots of the testing machine. A snapshot of the testing stand with a fixed sample for experimental research is presented in Fig. 2.

Fig. 2. Testing equipment

During the course of the research the compressive force and deformation was recorded, measured by resistance tensors, arranged in the longitudinal direction of the column on opposite sides of the web at the place of the biggest expected deflections. Post-critical equilibrium paths obtained from the conducted test, specifying the relationship between the load and the difference of the deformation P-$(\varepsilon_1$-$\varepsilon_2)$, made it possible to determine the value of the critical load and the evaluation of the structure in the low post-critical stage.

The inevitable inaccuracies occurring in the course of experimental research caused by different independent factors, such as imperfections geometry of the structure, testing stand design and implementation of loads and boundary conditions make it difficult to precisely determine the value of the critical load. In such cases, it is necessary to use methods of approximation, to estimate the value of critical load, based on the measurements carried out in experimental studies. In the presented study in order to assess the value of critical load, two independent methods of approximation were used: Koiter's method and the method of vertical tangent.

Application of the Koiter's method was based on the approximation of the post-critical equilibrium path, which describes the relationship between the load of the sample and the measured deformations differences on the opposite sides of the profile's walls. Experimentally determined post-critical equilibrium path, P-$|(\varepsilon_1$-$\varepsilon_2)|$ in the range of low post-critical state is approximated squared function. The value of the critical load is defined as the intersection of the function of approximation and vertical axis of the coordinate system of the post-critical characteristic of the structure P-$|(\varepsilon_1$-$\varepsilon_2)|$.

In the case of the vertical tangent method, the critical load value is also determined on the basis of post-critical equilibrium path, however estimation of the approximate critical force is based on a load to mean deformation characteristics. In this study, the value of deformation ε_{av} were based on the indications of strain gauges accordingly to the formula $(\varepsilon_1+\varepsilon_2)/2$. Post-critical equilibrium path P-ε_{av} was approximated by the vertical-linear function maintaining a constant value of the x-axis coordinate being equal to the extreme of the deformations (the coordinate point of inflection of the post-critical path). The critical load is determined by the coordinate of the ordinate axis of intersection of the approximation function with the equilibrium path of the post-critical structure characteristics.

In research carried out, a key determinant deciding about accuracy of the process of approximation was the coefficient of correlation R^2. The value of this coefficient determined the level of convergence of the approximation function with a selected range of the approximated experimental curve. For the applied process of approximation of the experimental post-critical equilibrium paths of the structure, the minimum value of the correlation coefficient of $R^2 > 0.95$ was established.

3.2. Numerical analysis

Simultaneously with the experimental studies, numerical analysis of stability and post-critical states using the finite element method were performed. In the process of numerical calculations commercial program ABAQUS® was used. The scope of conducted calculations included the analysis of critical and low post-critical state for up to 150% determined lowest critical load. Analysis of the critical state included the solution of linear analysis of the eigenvalue problem, which led to the designation of the critical load and the corresponding lowest form of buckling. In order to solve eigenvalue problem the use of conditions for extreme energy potential (the state of equilibrium corresponds to the minimum potential energy) was required. This means that for systems with a static second variation of the potential energy must be positive definite. The second stage included the calculation of non-linear static analysis, conducted on a model with initialized geometrical imperfection, corresponding to the lowest form of buckling, the value of the amplitude of 0.1 of the wall profile thickness. This allowed the determination of the post-critical equilibrium path of the structure, defining the dependence of the load and wall profile deflection in the normal direction P-w in the low post-critical range. Solution to the problem of geometrically nonlinear was performed using the incremental-iterative method of Newton-Raphson.

The process of discretization of the numerical model was carried out using shell elements, with 6 degrees of freedom at each node. The element type of designation S8R was used, which is a 8-node part, with the function of the shape of the second order and reduced integration. Reduced integration technique is one of the oldest techniques of the approximation of the solution of displacement and tension states in the element. Reduced integration allows to remove the false forms of deformation of finite elements by using higher order polynomial function in the description of the shape of the element [16]. In the process of discretization structural finite element mesh was used, taking the size of the element equal 2 mm. The adopted method of discretization ensured equitable distribution of the individual profile walls, leading to the size of the numerical model consisting of 5760 finite elements and 17585 nodes.

Finite element type, used in the process of discretization, was of a layered structure, which allows the definition of the laminate's structure by the thickness of the element. In the developed numerical model, by adopting the experimentally determined mechanical properties of the composite material-orthotropic material model in a plane state of tension was defined.

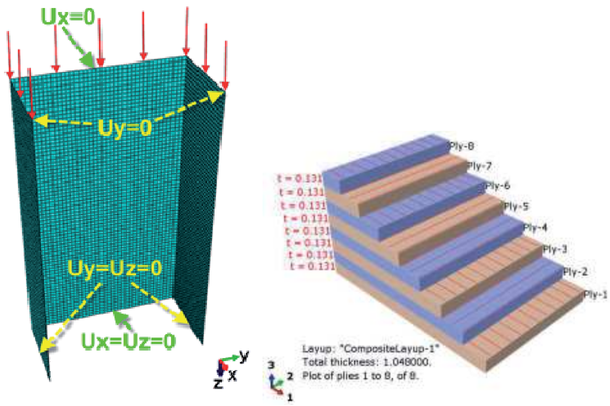


Fig. 3. Discreet model of the C-shaped cross-section column with the assigned boundary conditions

The boundary conditions assigned to the numerical model corresponded for the unhampered articulated support of the compressed composite columns, was shown in Figure 3. The implementation of boundary conditions was carried out by setting an zero displacement for nodes situated on the edges of the lower and upper column end-sections, respectively for the perpendicular directions to the plane of each wall. Furthermore the nodes placed in the edge of the lower end of the column were set to block the movements in the vertical direction. In addition, permanent displacement of the nodes placed in the edge of the upper end of the column was assigned by engagement of the displacement in the axial direction of the column. Numerical model was subjected to a distributed load, applied to the edge of the upper section of the column, ensuring an even squeezing in the axial direction.

4. Results

The experiment of axial compression of the thin-walled, channel sectioned column provided the necessary information that allowed the assessment of the actual deformation of the structure in a function of external load. The obtained results allowed to make qualitative and quantitative analyses of up-to-critical and critical state, based on the recorded parameters of the test. Identification of the critical state was based on the obtained form of buckling and the corresponding value of the critical load. Experimentally determined critical values formed the basis for the verification of the FEM numerical calculations.

The study of the critical up-to-critical state showed that the lowest value of the critical load corresponds to the local form of buckling of the structure manifested in the creation of a half-wave on individual walls and the web of the C-shaped profile. Obtained in the experimental study and numerical calculations lowest form of buckling of the structure was shown in Fig. 4.

a) b)

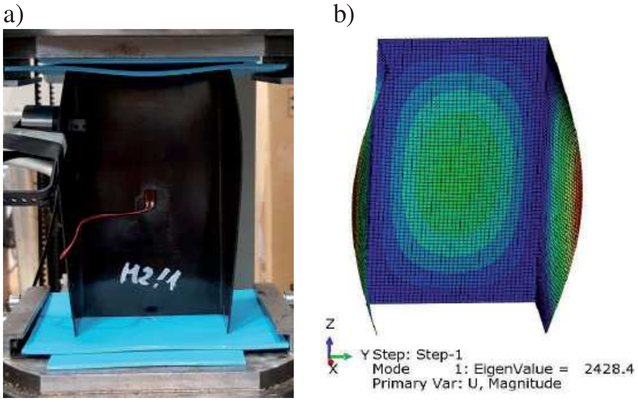


Fig. 4. The lowest form of buckling of the C-shaped cross section column: a) experimental studies, b) FEM calculations

Obtained in experimental studies and numerical calculations forms of deformation of the compressed column with C-shaped cross-section, shows the total qualitative compliance of structure buckling. Measurements of deformation with the use of resistance strain gauges enabled the determination of post-critical equilibrium path of the structure, specifying the dependence of the loading force from the average deformation $P–(\varepsilon_1–\varepsilon_2)$. The resultant characteristics formed the basis for determining the value of the critical load by means of two independent methods of approximation: Koiter's method and the method of vertical tangent. The key problem with this approach is the proper selection of the measuring range necessary to describe the post-critical path, which directly translates into the obtained results. In the case of inadequate selected of the approximation procedures, the critical loads differ significantly from the values estimated in the numerical simulations. Furthermore the selection of the range of approximation should be dictated by maintaining the highest possible correlation coefficient R^2, signifying keeping the sufficient accuracy adjustment of the approximating function to the experimental curve. From the simulations performed for the Koiter's method, it was specified that the range of the approximated experimental curve included partially experimental post-critical path, beginning at the point of inflection line of force-deformation curve, defined by the end of the experimental able to low-post-critical state, while maintaining a high correlation coefficient ($R^2 > 0.95$). Vertical tangent method was associated with a precise determination of the extreme deformation value (inflection point of the structures equilibrium path), which was approximated by a linear function with constant ordinate (vertical line).

In the case of the Koiter's method, experimental critical path in the form of P-$|w|$ was analyzed, where the deflection value was determined by formula $|w|\approx|(\varepsilon 1-\varepsilon 2)|$. Sequentially obtained curve was approximated by the polynomial of the third degree. The value of the critical load was in this method set by the intersection point of the resulting polynomial with abscissa axis. The obtained value of the critical load using the Koiter's method was 2564.5 N – Fig. 5.

In the case of the vertical tangent method, post-critical equilibrium path as P-ε_{av} was analysed. Average value of deformation was described with formula $\varepsilon_{av}=[(\varepsilon 1+\varepsilon 2)/2]$, as for which the extreme deformation was determined. The obtained point was used to approximate equilibrium path with tangent vertical. In this case, the critical load value was determined by coordinate of the load axis, in point of intersection of the resulting vertical line with the line of post-critical equilibrium path of the structure. Determined by the method of vertical tangent value of the critical load was 2504.06 N, as shown in Fig. 6.

Designated by the methods of approximation, the critical load was compared with the lowest value of eigenvalue, obtained in the numerical analysis of the critical state. The resulting numerical value of the critical force for the numerical model was 2428.4 N. The values of the critical load using all the test methods was summarized in Figure 7.

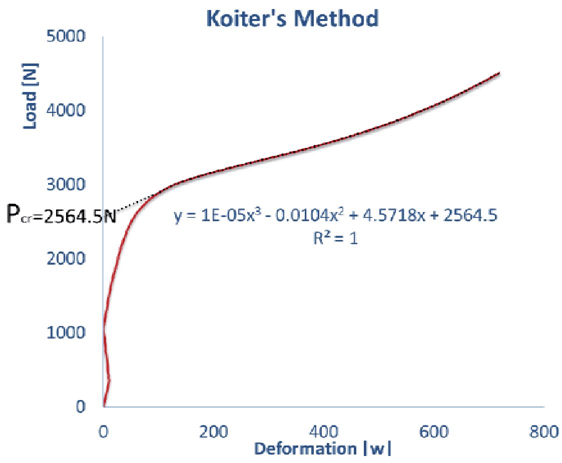


Fig. 5. Determination of the critical load using Koiter's method

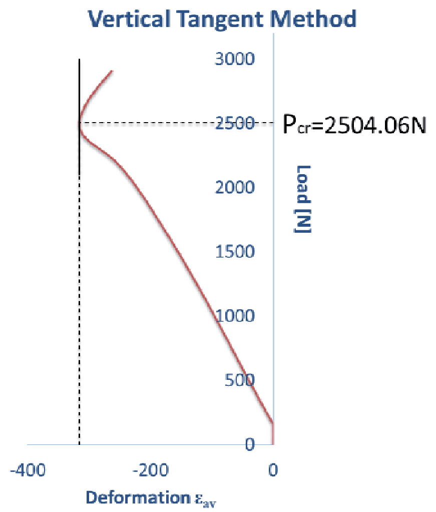


Fig. 6. Determination of critical load using the vertical tangent method

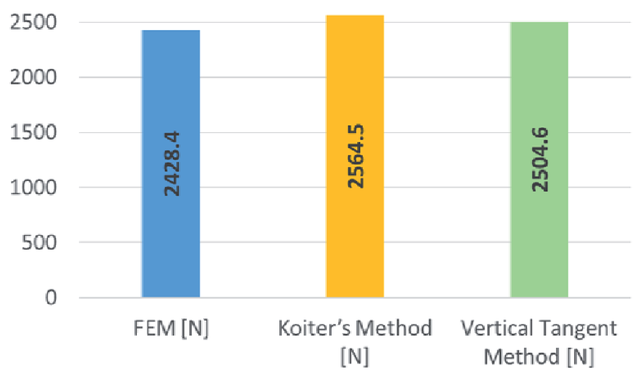


Fig. 7. The values of the critical load – the results of experimental and numerical analysis

Values of the critical force obtained by the numerical and approximation methods at which buckling of the thin-walled - channel section column occurs, is characterized by very high degree of alignment. The maximum difference in value between experimental and numerical methods, which occurs in case of the Koiter's method, equals 5.3% (for the method of tangent vertical difference indication equal 3%). It indicates a very high compatibility of quantitative test methods used, while confirming the correctness of the proposed procedure for determining the value of the critical load for real structure.

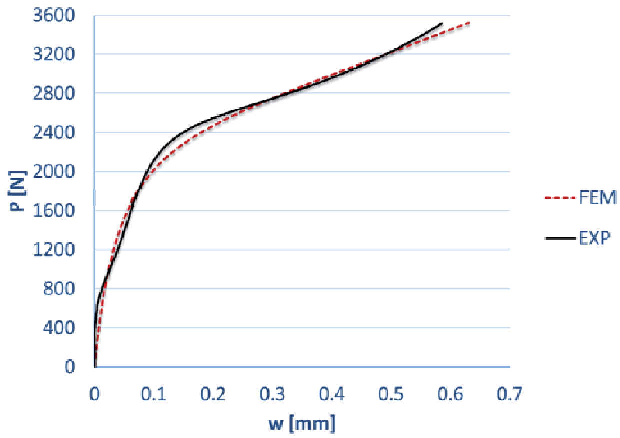


Fig. 8. Post-critical equilibrium paths FEM-experiment

In order to verify the selected test methods, post-critical equilibrium paths were compared *P-w* (compressive force - maximum deflection in a direction perpendicular to the wall profile) set in terms of low post-critical range. Shown in Fig. 8 experimental characteristics exhibits compliance of quantitative and qualitative with curve determined in the numerical simulations carried by finite elements method. The presented results confirm the adequacy of the developed numerical model enabling the simulation of critical and post-critical state of the channel section composite columns subjected to compression.

5. Conclusions

The thin-walled profile with a C-shaped cross-section subjected to axial compression process was analysed. One of the purposes of the study was to determine the value of the critical load, on the basis of the experimentally obtained post-critical equilibrium paths of the structure, with use of two independent methods of approximation: Koiter's method and the method of vertical tangent. The results of the analysis were compared with the value of the critical load determined by using finite element method. A very high compliance of critical load, designated by the each method was found – the maximum differences do not exceed 6%. This confirms the possibility of using the proposed procedure for determining the value of the critical load of a real structure. Precise determination of the critical load, in the case of thin-walled structures is one of the issues of enormous significance for operational reasons, it allows structure to protect against undesirable buckling.

Analysing the studies carried out, attention should be paid to the high sensitivity of the of approximation parameters which significantly affect the accuracy of the results. Applies in particular to proper selection of the range of approximation and the need to ensure a high correlation coefficient R^2, ensuring compliance of the experimental design characteristics witch the function approximating.

The carried out analysis provided the high qualitative and quantitative compliance of the experimental results and the numerical calculations. This applies to, buckling of the structure, the critical load value, as well as post-critical equilibrium paths P-w in the low post-critical range. The obtained results provide the important information concerning the modelling techniques of the thin-walled structures made of composite materials, while confirming the adequacy of the numerical models developed both in the calculation of eigenvalue problem, as well as non-linear static analysis in the post-critical range.

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