

# Application of Voronoi diagrams in contemporary architecture and town planning

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Modern design methods rely increasingly on understanding the nature of processes and principles of self-organization of biological structures and their representation using mathematical models that may apply in technology, including architecture. As a result, bionic design elements play a more meaningful role in shaping contemporary architecture and urban planning. The development of computer technology has made it possible to create more complex and complicated structures and surfaces inspired by natural forms. The inspiration for the discretization of the surface, using the Voronoi diagram, as seen in the honeycomb structure or the dragonfly wing, is increasingly applied in shaping the elevation of contemporary buildings. As a mathematical problem, the division of space fascinates scientists as well as architects. Consequently, architects use the spatial Voronoi cells also in shaping the structural forms. Today Voronoi diagrams are an important source of inspiration for architects and urban planners as a surface discretization method and a way of creating structural elements or spatial forms and as flooring patterns in urban projects. The use of mathematical models represent the structure and organization of the forms found in nature, which is increasingly used in the multidisciplinary architectural design. The design of the structures and elements both in architecture and urban planning using methods of computational geometry makes new opportunities for architectural and urban projects as seen by the Voronoi tessellation.

**Key words:** bionic design, Voronoi diagram, architecture, town planning

## Introduction

Modern design methods rely on understanding the processes of nature, the principles of self-organization of biological structures and their representation using mathematical models. The bionic design plays a more meaningful role in shaping contemporary architecture and urban spaces. The Voronoi diagram is an example of leading trends in architecture design. The Voronoi Tessellation describes a system of the self-organization of biological structures visible on the wing of a dragonfly, the turtle shell, honeycomb or the shell of a sea urchin. (Fig. 1)

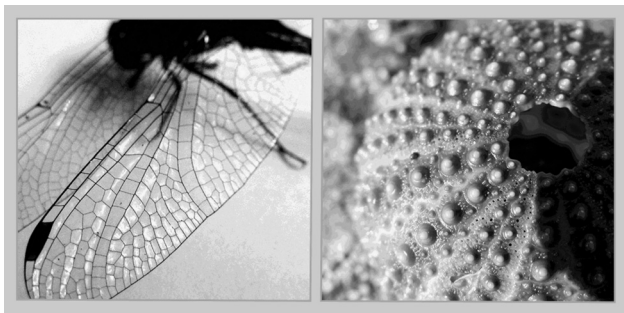


Figure 1. Example of Voronoi diagram found in nature in dragonfly wing (left) or sea urchin shell (right)

The Voronoi diagram is a graph consisting of cells created from ovules, the edges and nodes of the Voronoi diagram (an equidistant point from at least three centers [1]). The Voronoi diagram is built up of segments and half lines which constitute the edges of Voronoi areas for each of the centers. The Voronoi area is defined for each center as a set of points on the plain which are closest to a given center. The Voronoi diagram is defined for a given set of  $n$  points as plain divided into  $n$  areas in such a manner that each point in any cell is closer to a specific point from a set of  $n$  points than the remaining  $n-1$  points. In two-dimensional space, the Voronoi cells are convex polygons, in three-dimensional space they form convex polyhedral. The Delaunay Triangulation is a dual graph to the Voronoi diagram. (Fig. 2)

A progressive algorithm, divide and conquer, or Fortune can be used to construct the Voronoi diagram. [2] The Voronoi diagrams allow for the creation of an optimal grid of junctions, sparking a broad interest among researchers in the shaping of architecture, construction, and public spaces. Thanks to the development of digital tools used in the design process the Voronoi diagrams are used in modern architecture and urban planning. A multitude of computer programs allows (for e.g. grasshopper or software created for projects) for a broad spectrum of shaping possibilities and the innovative search of new forms leads to the creation of interactive applications and algorithms the

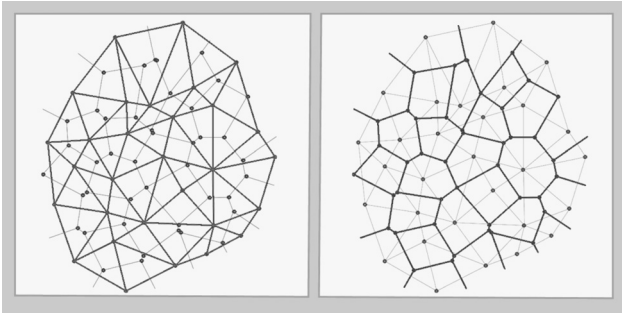


Figure 2. Delaunay triangulation (left) and Voronoi tessellation (right) as a dual graph

constructing the optimal structures. The swift development in this field justified conducting a study based on the analysis of examples to determine the possibilities of the Voronoi diagrams in architecture and urban planning. The article was conducted as a case study as a research applied in architecture. [2]

### Examples of the application of the Voronoi diagram

Voronoi diagrams are also used in shaping the urban planning concepts. The project of redeveloping the Glorieta Juan Carlos I in Spain is an example of this application. The ESC Design Studio project proposed the creation of a self-sufficient space thanks to photovoltaic elements equipped with artificial fog systems. The space planning of the square has been generated with custom software that implements a Voronoi algorithm to divide the space into a walkway and activity areas. In effect, designers get a dynamic pattern that runs through the space among the programmed activity areas and performance spaces. The interactive software was also used during public consultations, allowing for the adaptation of the concept to the societal needs.

The inspiration with Voronoi diagrams are visible in the conceptual Aldgate Aerial Park design in London. The Matsys project featured the creation of a public space over the streets of the city, at the border of the old and new town during the 2012 Summer Olympics. Open spaces, gardens, theaters, etc. were placed in the individual cells of the Voronoi diagram. (Fig. 3)

It is also worth noting that the Voronoi diagrams are used in searching the immediate neighborhood, determining the function of the position (vertices), finding the largest empty circle (cell), and path planning (edges). [3] This is why Voronoi diagrams could be used in urban planning in determining the optimal deployment of infrastructure or functional separation.

The shaping of the architectural and structural forms using the Voronoi Diagrams is one of the most important trends in seeking new forms of expression in architectural design. In shaping contemporary architecture, the Voronoi

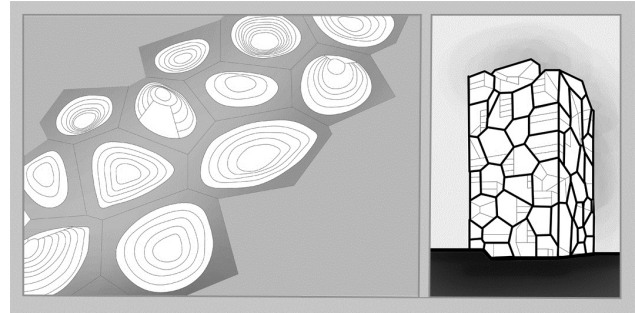


Figure 3. Example of application of Voronoi diagrams in urban spaces in the conceptual Aldgate Aerial Park design in London, designed by Matsys (left) and in 3d structures in the conceptual project Vertical Village, designed by architects Yushang Zhang, Rajiv Sewtahal, Riemer Postma and Qianqian Cai (right)

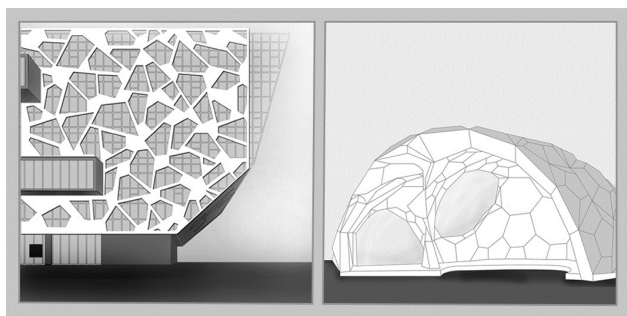
diagram is a synergetic solution for a new generation in architecture. Based on Voronoi cells, the discretization of surface and the shaping of spatial forms lead to the creation of 3D structures.

As an effective way of dividing surfaces, the Voronoi diagrams are used for the discretization of the structural architectural surface. The Voronoi Tessellation is used both in the roofing design and architectural façade objects.

The roofing of the WestendGate public space in Frankfurt am Main in Germany is an interesting example thereof. The Just Burgeff architekten and a3lab design was completed in 2010 as part of the high-rise Marriott Hotel renovation. The arboreal support roofing project generated divisions of the roof in the form of Voronoi diagrams; these became optimized due to the weight of the finite element method. The carried out research paved the way for the interdependence of form and structure. As a result, the structure was made of same diameter steel pipes with a three-layer wall thickness. The application of arboreal supports allowed for a reduction in the number of supports and an optimal roof surface carrying capacity of 1000 sq. m. with a variable height reaching up to 14 meters. In this case, the supports were not located in the center of the cells. The curved roof surface was filled with transparent, pneumatic ETFE foil cushions fitted to each Voronoi cell.

The Greenhouse Botanical Garden project in Groningen, Switzerland is another example thereof. The project features a harmonious connection of architecture and the surrounding forest. As a result, a Voronoi diagram steel structure was proposed. The arboreal supports were placed in the center of the diagram thereby providing maximum support for the roof, while minimizing the number of supporting poles. This resulted in a form similar to a tree, which combined with the glass facade allowed for a smooth blending of the object into the landscape.

The Voronoi diagrams are also used in shaping the grates, as featured on the Campus Restaurant and Event Space project in Stuttgart. The Barkow Leibinger Architects



**Figure 4.** Example of the application of the Voronoi diagrams as an exterior structure on the Alibaba Headquarters facade in Hangzhou, designed by Hassell Architects (left) and as a biological principle of creating forms and structural elements in Research Pavilion 2011, designed by ICD and ITKE at Stuttgart University (right)

design in collaboration with Werner Sobek Ingenieure assumed a pentagonal roof division forming triangular shape elements, the basic structure of which (beams and columns) is steel-made. A wooden grate complements the construction system, the divisions of which were shaped according to the Voronoi diagram.

The Voronoi Tessellation is also used in the discretization of the facade. In the Melbourne Recital Centre project the architect Ashton Raggatt McDougall proposed curtain wall facades inspired by a honeycomb. This method of discretization of the surface was also used to create facade tile divisions. A similar example is the Alibaba building Headquarters in Hangzhou in China. Voronoi diagrams are used to create a structure which includes decking atrium combined with the outer structure of the building. (Fig. 4)

The Voronoi diagrams can also be used in the discretization of spatial forms. As a result, it is possible to create objects with a natural and remarkable design inspired by forms found in nature or applying natural self-organization systems of biological structures in the classical geometrical forms.

Times Eureka Pavilion designed on the RHS Chelsea Flower Show in 2011 is an interesting example of an open pavilion. The NEX Architecture studio project authored by landscape architect Marcus Barnett and structural engineer Buro Happold is based on an algorithm that simulates the process of growing trees (L-systems) and Voronoi diagrams. This way the pavilion underlines the natural ecosystem interdependence of science and technology. The formation of the pavilion structure was inspired by the principle of natural structures, their development and growth of plants. The object was based on a digital model using numerical control machines (Computerized Numerical Control) for 5-axis milling of a wooden structure and laser cutting of plastic elements. [4] For computer modeling purposes, Processing and Grasshopper programs were used. [4]

The use of Voronoi diagrams in architecture is being increasingly researched in university institutes. This subject

is raised in student projects, inter alia, by Manuel Fabian Hartmann of the University of Innsbruck. The design process was conducted on the basis of the “SMART form” digital tool made available by Al Fisher from Buro Happold. The form of the object was generated by taking into account the functional and locational conditions. The generated models were optimized in Clemens Preisinger’s program „Karamba”. Double curved elements were “realized by an approximation with several curved shapes simply as D-Forms,.. [5] The individual elements were generated based on Voronoi diagrams, which allowed for the optimization of the object’s supporting structure. As a result, a double curved structure was designed with 7 x 5 m dimensions and a height of 2.6 m composed of 30 pieces of curved plywood and thickness of 6.5 mm. The light, wooden structure has circular openings that provide outlooks on the surrounding nature.

Research Pavilion designed by the Institute for Computational Design and the Institute of Building Structures and Structural Design at the Stuttgart University is yet another example. The object was inspired by the sea urchin shell. In the shaping process an algorithm was used that simulates the natural forming processes with the Voronoi principles. The resulting forms are verified in terms of space and strength, as well as taking the production capacity into account. Particular attention in the design process was paid to the interdependence of form and structure of the building. As a result, prefabricated wooden panels manufactured using numerically controlled machines (robotic fabrication system) were proposed. This in turn made it possible to create a lightweight construction with a relatively high carrying capacity. The panel joints were designed to transmit normal and shear stresses without the bending effect. A formation of geometrically effective form was possible due to an integrated connection system. (Fig. 4) The structure of the object possesses the characteristics of a biological structure, i.e. diversity, anisotropy and hierarchy. [6]

The use of Voronoi diagrams in discretization of spatial forms was put to a test while shaping the New Harmony Grotto in a research project led by Metalab Architecture + Fabrication group in cooperation with Gerald D. Hines, Mrs. Jane Blaffer Owen, professors Andrew Vrana, Joe Meppelink, Ben Nicholson and a group of students from the College of Architecture in Houston, Texas. The object was to serve as a permanent landmark on the University of Houston campus. The project uses digital technologies for 3D modeling, three-dimensional scanning, and producing each of the elements in the Computer Numerical Control process with the use of laser cut stainless steel. The object’s structural surface divisions were generated as minimal surfaces, based on the Voronoi diagrams, constituting natural formation processes. The structural grid is made of steel, each frame constructed of metal sheets was linked to a concrete foundation.

The spatial Voronoi cells are being increasingly implemented in today's concepts. The ACME group's Un Memorial is one example that won third prize in the competition in the United Nations memorial project. The crux of the project was based on the Voronoi tessellation where the structure of the UN as individual nations was mapped by the very same cells. The Voronoi Tessellation reflects the „collective nature of the UN's identity,„ [7]. The facility includes exhibition spaces, offices and restaurants.

The Vertical Village concept authored by architects Yushang Zhang, Rajiv Sewtahal, Riemer Postma and Qian-qian Cai is a similar example. The concept involves improving the efficiency of the building by using geometrical elements in the form of spatial Voronoi diagrams. As a result, it was possible to obtain the optimum design of the object. (Fig. 3) Both the residential units and all other rooms were generated in accordance with the algorithm. Due to the division of individual modules of the project, the construction of such facilities is to be relatively easy. Consequently, the concept features a number of sophisticated aspirations coupled with a balanced development.

The application of Voronoi Diagrams in art installations is also interesting. The Le Voronoi project authored by the Collectif Time studio could serve as an example of such application. The artwork assumed placing an egg-shaped sculpture made of mirrors on the tree. The divisions in the elliptical form with dimensions of 6 x 4 meters, were generated based on the Voronoi diagrams. The individual mirrors are located in such a way to achieve the kaleidoscope effect.

The Honeycomb morphologies, which is the manifold installation designed by Matsys, is another example of a wall design. This installation is "simply a natural material system which have been developed through evolution as a form grown by living organisms. The Honeycomb Morphologies Project is based on the desire to form an integrated and generative design strategy using a biomimetic approach to architectural design and fabrication." [8] The artwork is the result of a research that focused on the possibilities of the Voronoi diagram usage in the development of a biomorphic structure, assuming there is a way to pre-fabricate elements using new technologies.

One of future approaches to using the Voronoi tessellation in architecture is the research for emergent structures. A project called Chrysalis III by Matsys is an example of the search for emergent forms based on the self-organization of cells. The project uses an algorithm that allows for a formation of a structure in which the Voronoi cell attempts to find a more balanced packed and find relaxed structure of the surface. „Each cell is composed of two parts: a cone-like outer surface made from cherry veneer and a non-planer inner plate made from poplar veneer that stresses the outer cone into shape." [9] Cells components are "unfolded flat in the digital model, digitally fabricated, and hand assembled." [9]

## Conclusions

The inspiration with nature is an important aspect in the design of architecture. It is particularly important to design bionic structures based on the formation of biological structures. The contemporary line of thought is to understand the processes occurring in nature and describing them appropriately in order to the apply these models in technology, including architecture. The development of digital tools currently used in architecture allowed for the portability of designs found in nature to the spatial and structural surface shaping. New methods change the way people work in architectural design. Consequently, algorithms simulating natural formation processes are increasingly more used in the field, shaping principles of the Voronoi diagrams. Structures based on the Voronoi tessellation, especially in the emergent structures, are a way of a synergetic solution for a new generation in architecture. The multitude of the Voronoi diagram implementation possibilities in the development of architecture, in particular the structural forms and structural elements, allows for an optimal design and a variety of spatial forms. The applicability of components of Voronoi diagrams in contemporary architecture, Urbanism and planning are shown in table 1. The design of the structures and elements both in architecture and urban planing using methods of computational geometry creates new opportunities for architectural and urban projects as seen by the example of Voronoi tessellation.

**Table 1. Possibility of application Voronoi diagrams in architecture, urban design and town planning**

Elements of Voronoi diagram		centres	edges	2D cells	3D cells
Application					
Town Planning		centre-forming points	route	region	X
Urban design		dominant	route or edges of specify area	activity area	small architecture
Architecture	Spatial form	points of areas of illumination	Structural element	surface	Structural form
	Facade divisions	points of areas of natural lightening or way of creation the divisions with adding the centres	edges of illumination	panels	X
	Structural surfaces	creation the divisions with adding the centres	structural element	panels	X
	Spatial structure			X	Structure

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